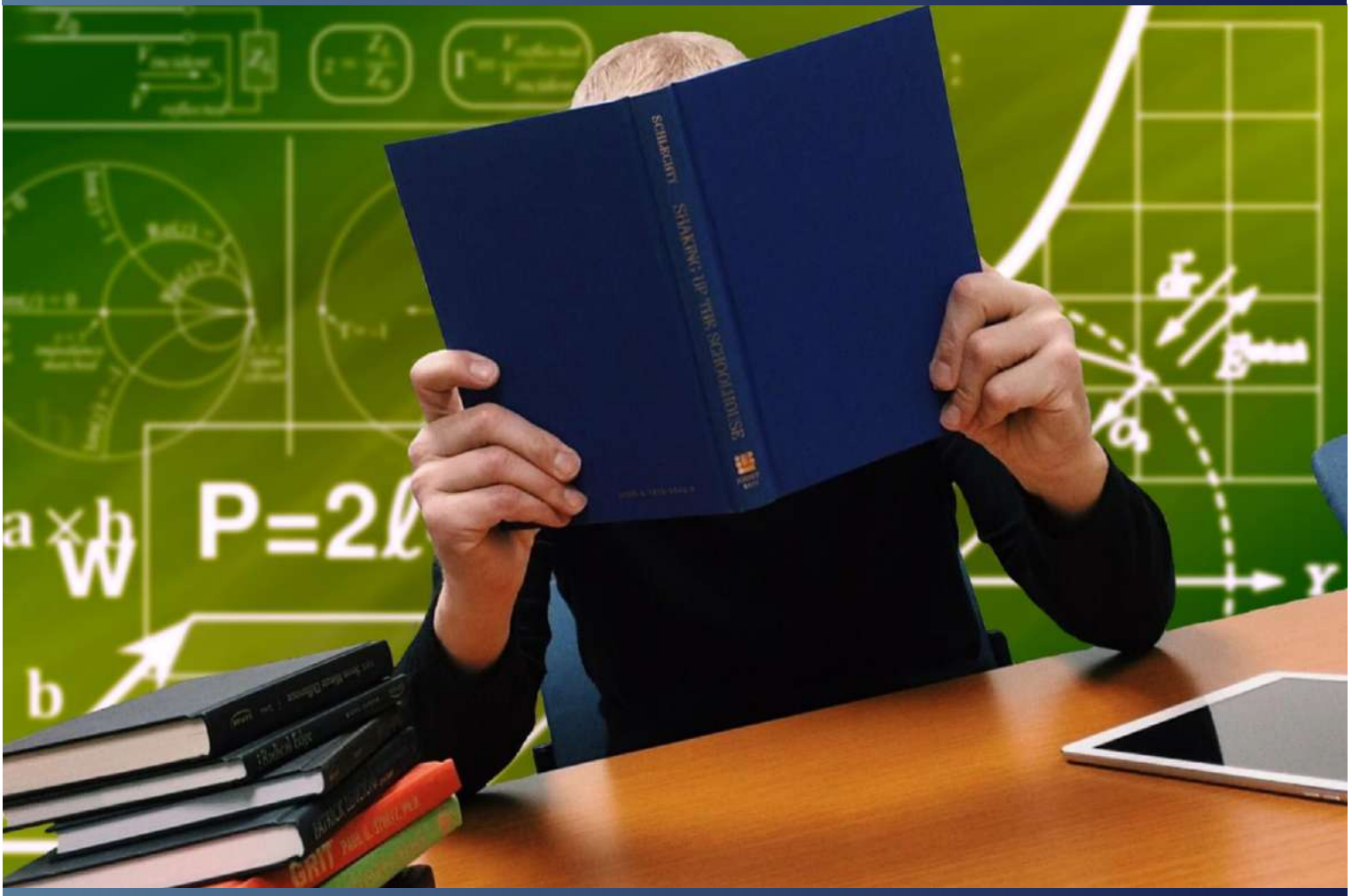
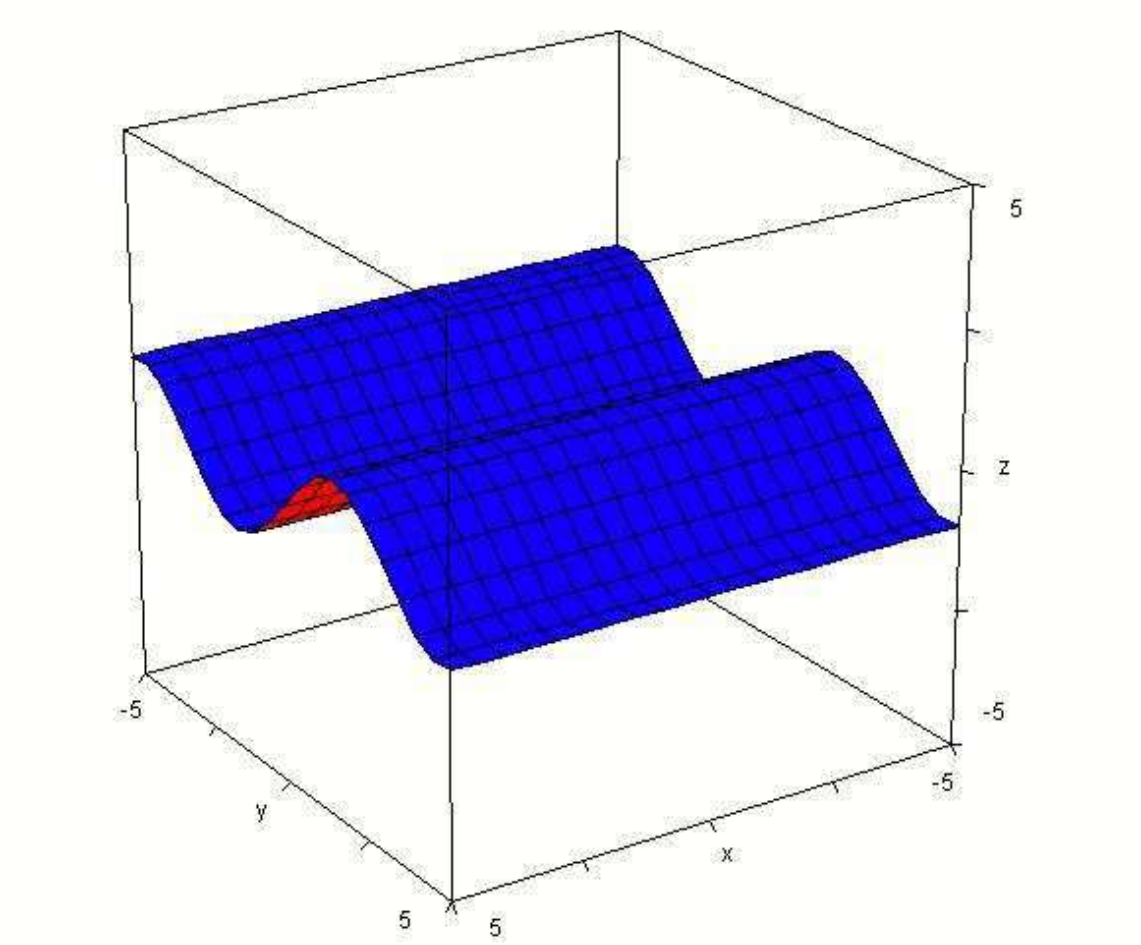
Ejercicios y Talleres



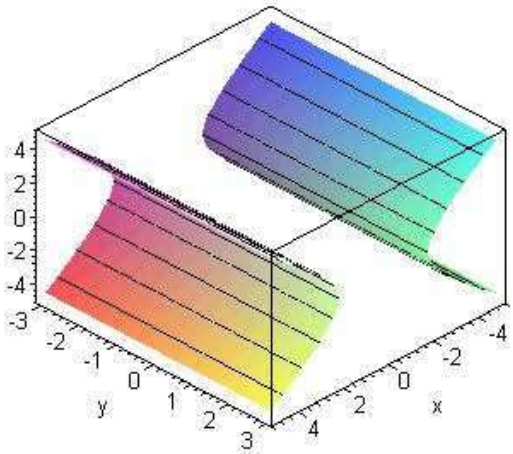
puedes enviarlos a
klasesdematematicasymas@gmail.com

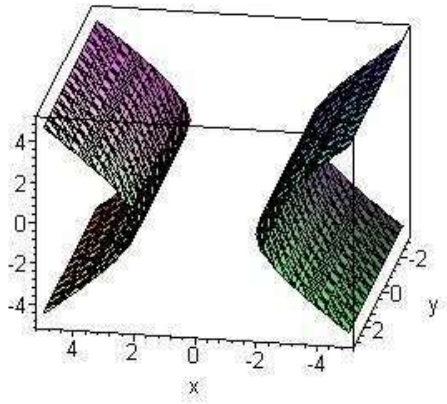
TALLER CÁLCULO VECTORIAL
SUPERFICIES

1) Grafique la superficie:
   a) Z = sen y

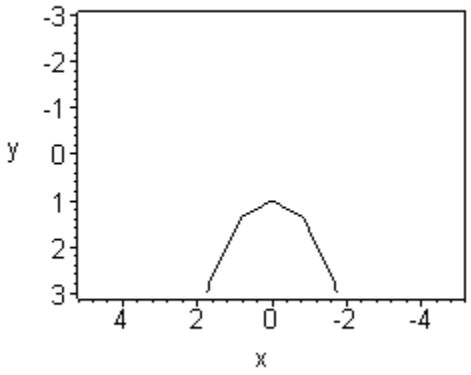


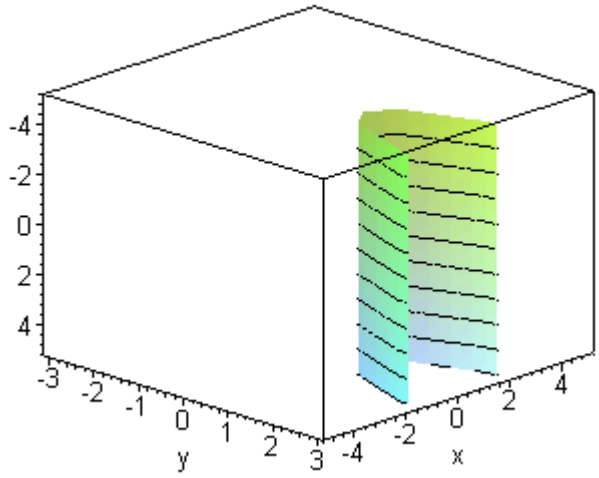
   b) $X^2 - z^2 = 4$





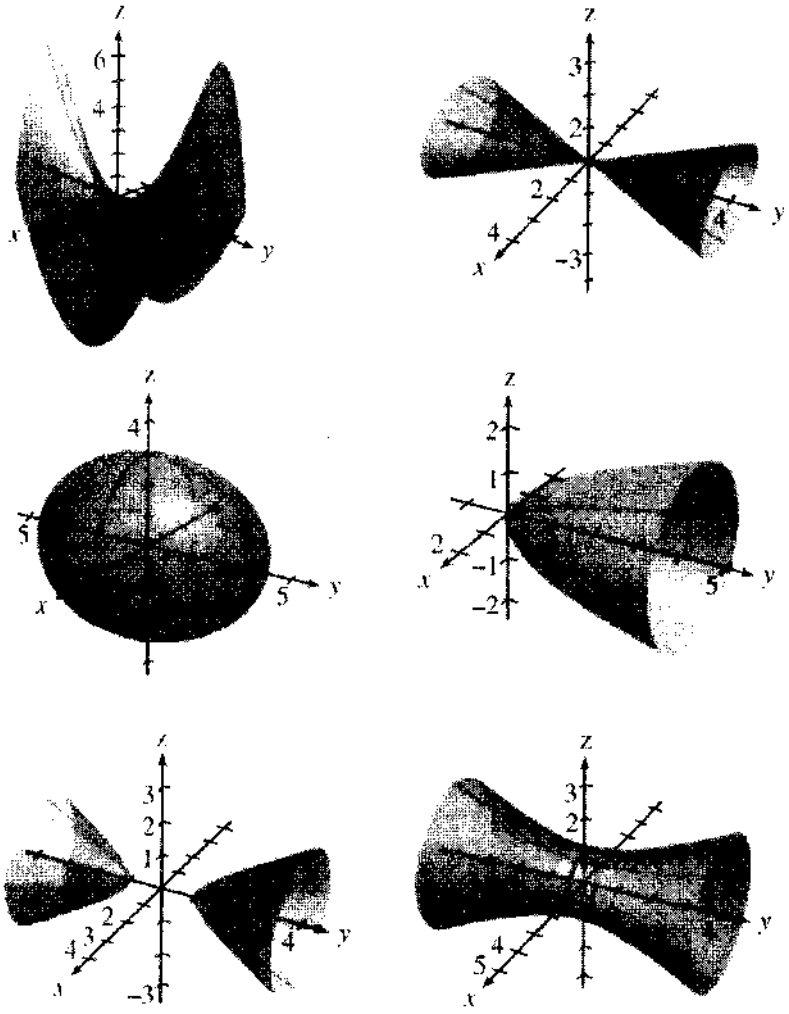
c) Y = cosh x





2)

En los Ejercicios 1-6, asociar cada ecuación con su gráfica.



1. $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} = 1$
2. $15x^2 - 4y^2 + 15z^2 = -4$
3. $4x^2 - y^2 + 4z^2 = 4$
4. $y^2 = 4x^2 + 9z^2$
5. $4x^2 - 4y + z^2 = 0$
6. $4x^2 - y^2 + 4z = 0$

Las gráficas son contadas de izquierda a derecha y de arriba abajo.

Gráfica No. 1. Ecuación: $4x^2 - y^2 + 4z = 0$

Gráfica No 2. Ecuación $y^2 = 4x^2 + 9z^2$

Gráfica No 3. Ecuación: $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{9} = 1$

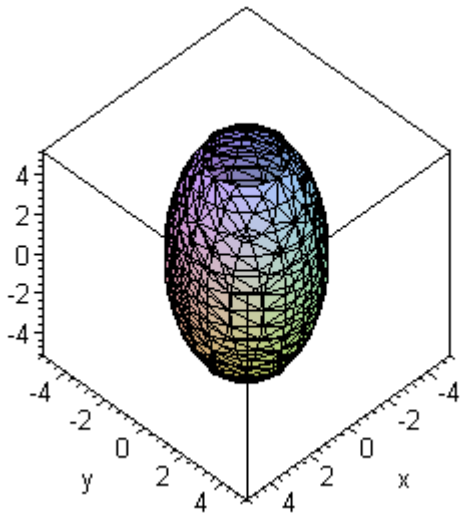
Gráfica No 4. Ecuación: $4x^2 - 4y + z^2 = 0$

Gráfica No 5. Ecuación: $15x^2 - 4y^2 + 15z^2 = -4$

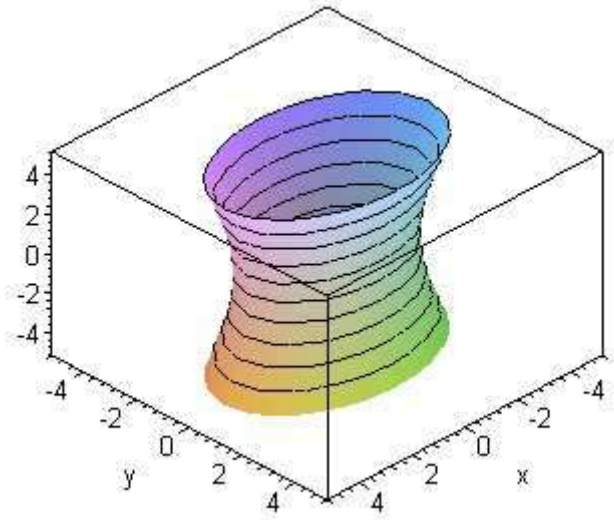
Gráfica No 6. Ecuación: $4x^2 - y^2 + 4z^2 = 4$

3) Grafique la superficie:

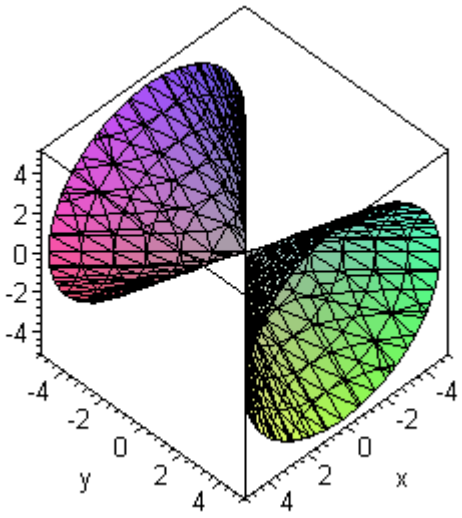
a) $4x^2 + 9y^2 + z^2 = 36$



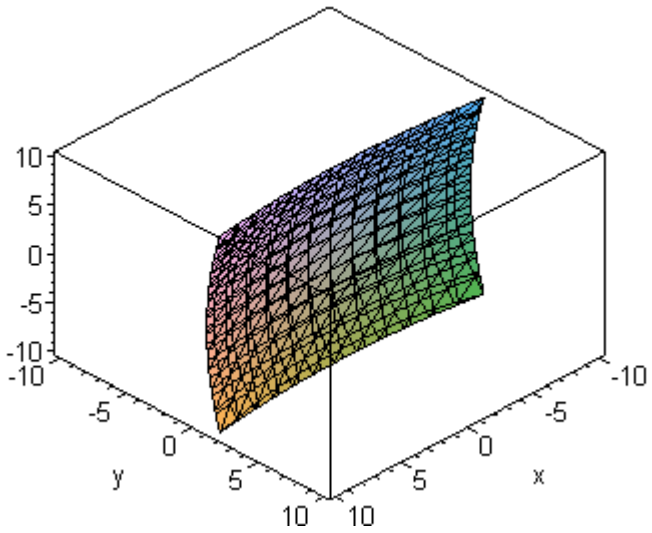
b) $4x^2 + 9y^2 - z^2 = 36$



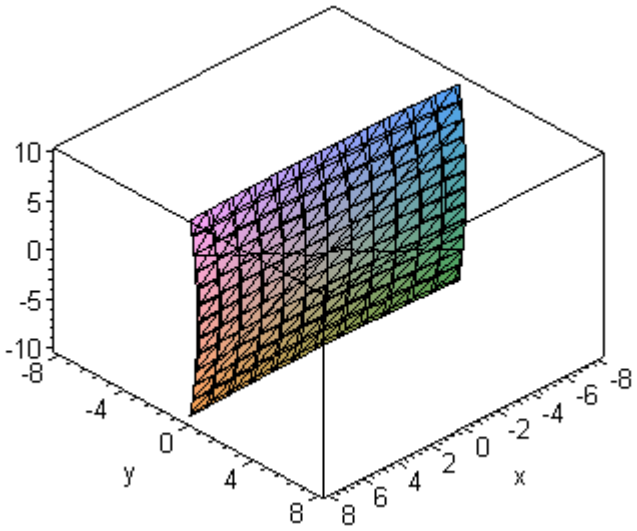
c) $X^2 = y^2 - z^2$



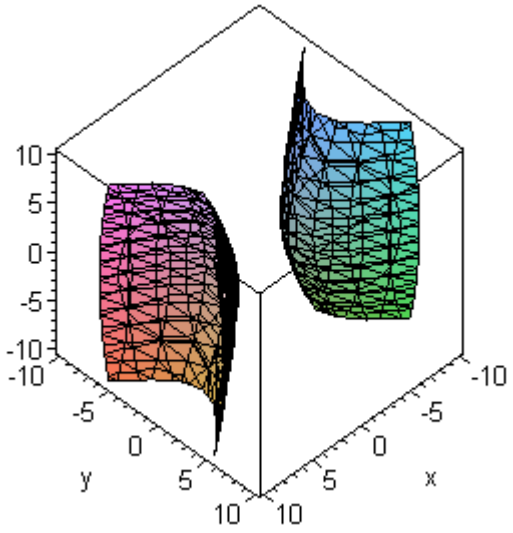
d) $X^2/36 + z^2/25 = 4y$



e) $X^2/36 - z^2/25 = 9y$

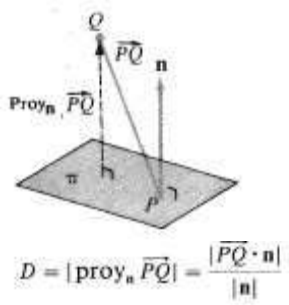


f) $4x^2 – 9y^2 – z^2 = 36$



4. Pruebe que la distancia entre el plano ax + by + cz = d y el punto (x_0, y_0, z_0) está dada por:

$$D = \frac{|ax_0 + by_0 + cz_0 - d|}{\sqrt{a^2 + b^2 + c^2}}$$



La distancia de un Punto Q a un plano está dada por:

$$D = \left| proy_n PQ \right| = \frac{\left| PQ \cdot n \right|}{\left| n \right|}$$

Siendo P un punto del plano, n el vector director del plano y Q un punto fuera del plano.
Para el punto p se tiene, y=y, z=z,

$$ax = d - by - cz$$

$$x = \frac{d - by - cz}{a}$$

El punto P es $\left(\frac{d - by - cz}{a}, y, z \right)$. El punto Q es $\left(x_0, y_0, z_0 \right)$.

El vector PQ es $PQ = Q - P = \left(x_0, y_0, z_0 \right) - \left(\frac{d - by - cz}{a}, y, z \right)$

$$= \left(x_0 - \frac{d - by - cz}{a}, y_0 - y, z_0 - x \right)$$

$n = \left(a, b, c \right)$ Vector director del plano

$$D = \left| proy_n PQ \right| = \frac{\left| PQ \cdot n \right|}{\left| n \right|} = \frac{\left| \left(x_0 - \frac{d - by - cz}{a}, y_0 - y, z_0 - z \right) \cdot \left(a, b, c \right) \right|}{\sqrt{a^2 + b^2 + c^2}}$$

$$D = \frac{\left| \left(x_0 - \frac{d - by - cz}{a} \right) \cdot a + \left(y_0 - y \right) \cdot b + \left(z_0 - z \right) \cdot c \right|}{\sqrt{a^2 + b^2 + c^2}}$$

$$D = \frac{\left| \left(ax_0 - d + by + cz \right) + \left(by_0 - by \right) + \left(cz_0 - cz \right) \right|}{\sqrt{a^2 + b^2 + c^2}}$$

$$D = \frac{\left| ax_0 - d + by + cz + by_0 - by + cz_0 - cz \right|}{\sqrt{a^2 + b^2 + c^2}}$$

$$D = \frac{\left|ax_0 - d + by_0 + cz_0\right|}{\sqrt{a^2 + b^2 + c^2}}$$ Queda demostrado