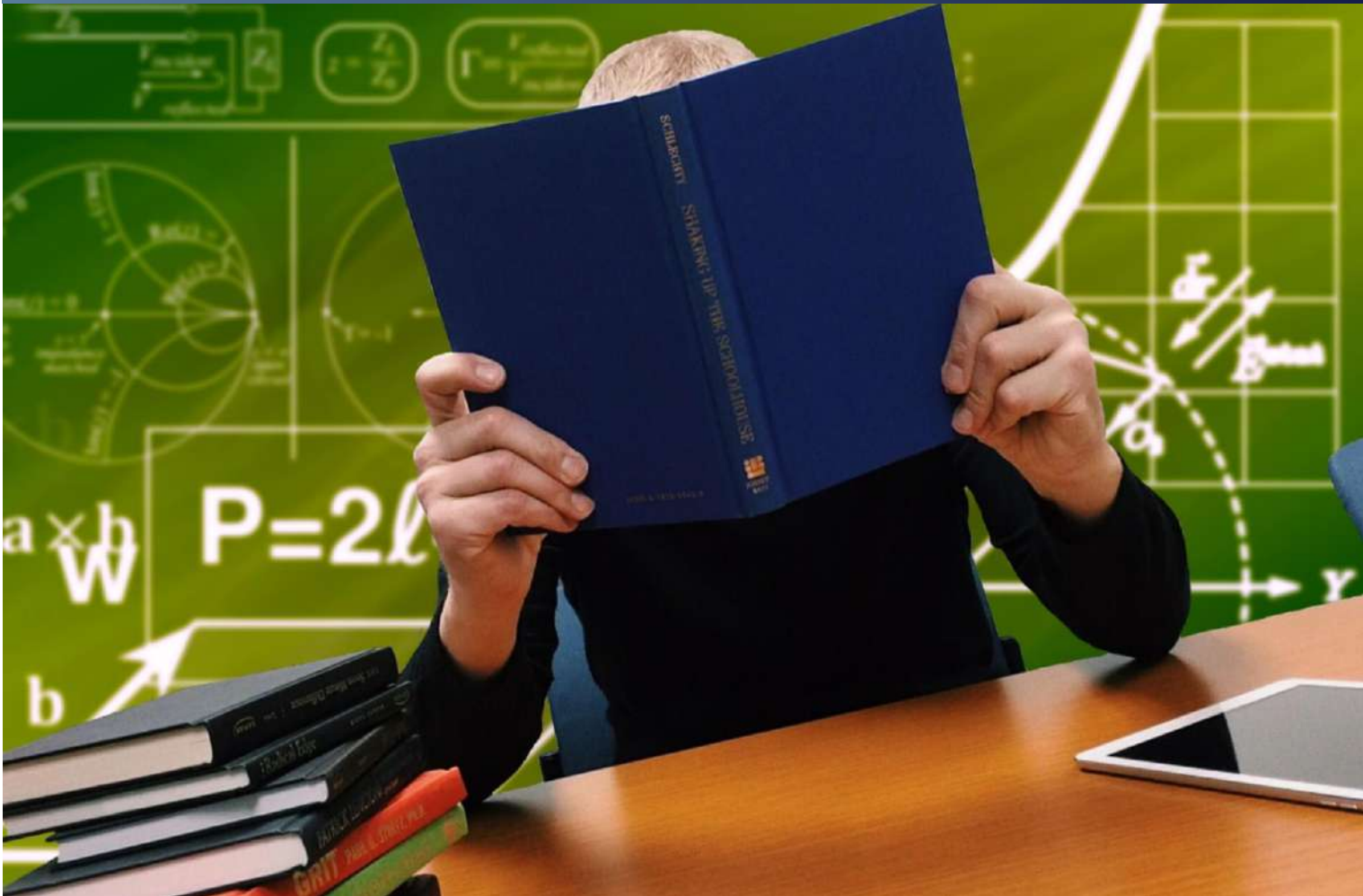


Ejercicios y Talleres



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Taller: Método de discos y arandelas

En los ejercicios 1 y 2, determine los volúmenes de los sólidos generados al hacer girar las regiones acotadas por las rectas y curvas alrededor del eje x .

1. $y = \sqrt{x}$, $y = 0$, $x = 4$.

2. $y = x^2 + 1$, $y = 0$, $-1 \leq x \leq 2$.

3. Determinar el volumen del sólido resultante al hacer girar la región comprendida entre la recta $y = 4$ y la curva $y = x^2$, entre $-2 \leq x \leq 2$, alrededor de la recta $y = 4$.

4. Calcule el volumen del sólido generado al hacer girar la región acotada por $y = x$ y $y = x^2$, alrededor de cada eje coordenado utilizando el método de arandelas.

En los ejercicios 5 y 6, determine el volumen de los sólidos generados al hacer girar las regiones alrededor de los ejes dados.

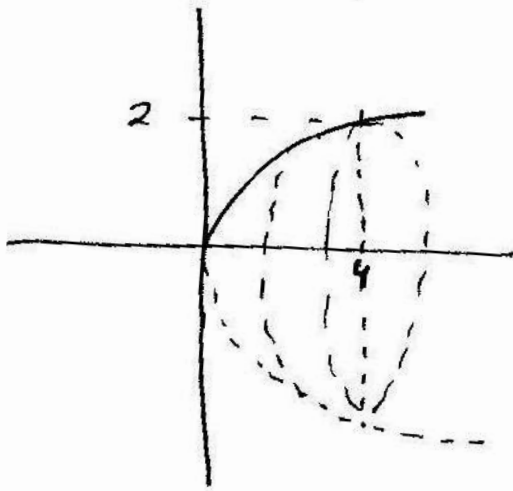
5. La región acotada por $y = \sqrt{x}$, $y = 2$, $x = 0$ alrededor de

- a. El eje x .
- b. El eje y .

6. La región acotada por $y = \sqrt{x}$ y $y = \frac{x^2}{8}$ alrededor de

- a. El eje x .
- b. El eje y .

$$1. \quad y = \sqrt{x} \quad y = 0 \quad x = 4$$



$$dV = \pi r^2 dh$$

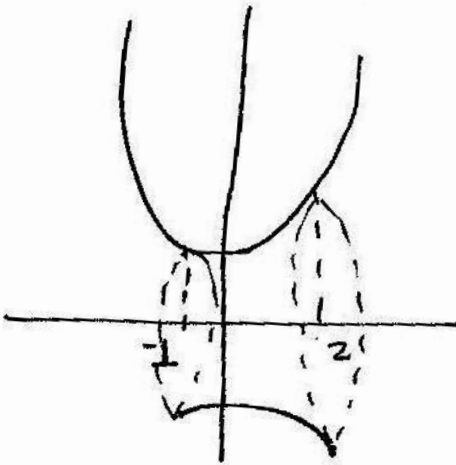
$$dV = \pi (\sqrt{x})^2 \cdot dx$$

$$dV = \pi x dx$$

$$V = \int_0^4 \pi x dx = \left. \frac{\pi x^2}{2} \right|_0^4$$

$$V = \frac{\pi}{2} (4^2 - 0^2) = 8\pi$$

$$2. \quad y = x^2 + 1 \quad y = 0 \quad -1 \leq x \leq 2$$



$$r = x^2 + 1$$

$$dV = \pi r^2 dh$$

$$dV = \pi (x^2 + 1)^2 dx$$

$$V = \int_{-1}^2 \pi (x^4 + 2x^2 + 1) dx$$

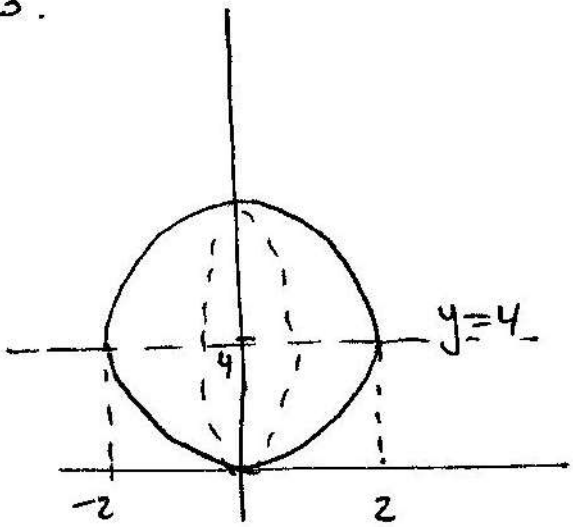
$$V = \pi \left(\frac{x^5}{5} + \frac{2x^3}{3} + x \right) \Big|_{-1}^2$$

$$V = \pi \left[\left(\frac{2^5}{5} + \frac{2 \cdot 2^3}{3} + 2 \right) - \left(\frac{(-1)^5}{5} + \frac{2(-1)^3}{3} + (-1) \right) \right]$$

$$V = \pi \left[\frac{32}{5} + \frac{16}{3} + 2 + \frac{1}{5} + \frac{2}{3} + 1 \right]$$

$$V = \pi \left[\frac{33}{5} + \frac{18}{3} + 3 \right] = \pi \left[\frac{33}{5} + 9 \right] = \frac{78}{5} \pi$$

3.



$$r = 4 - (x^2)$$

$$r = 4 - x^2$$

$$dV = \pi r^2 dx$$

$$dV = \pi (4 - x^2)^2 dx$$

$$dV = \pi (16 - 8x^2 + x^4) dx$$

$$V = \int_{-2}^2 \pi (16 - 8x^2 + x^4) dx$$

$$V = \pi \left(16x - \frac{8x^3}{3} + \frac{x^5}{5} \right) \Big|_{-2}^2$$

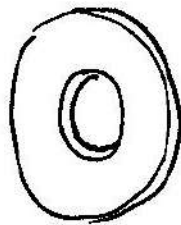
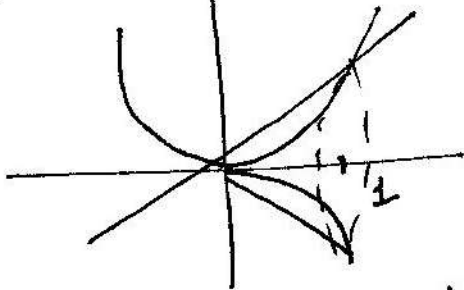
$$V = \pi \left[\left(16 \cdot 2 - \frac{8 \cdot 2^3}{3} + \frac{2^5}{5} \right) - \right.$$

$$\left. \left(16 \cdot (-2) - \frac{8(-2)^3}{3} + \frac{(-2)^5}{5} \right) \right]$$

$$V = \pi \left[32 - \frac{64}{3} + \frac{32}{5} + 32 - \frac{64}{3} + \frac{32}{5} \right] = \pi \left[64 - \frac{128}{3} + \frac{64}{5} \right]$$

$$V = \frac{512}{15} \pi$$

4) $y = x$ $y = x^2$
alrededor de x



$$dV = \pi (r_{ext}^2 - r_{int}^2) dh$$

$$r_{ext} = x$$

$$r_{int} = x^2$$

$$V = \int_0^1 \pi (x^2 - (x^2)^2) dx = \int_0^1 \pi (x^2 - x^4) dx$$

$$V = \pi \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_0^1 = \pi \left(\frac{1}{3} - \frac{1}{5} \right) = \frac{2}{15} \pi$$

Alrededor de y

$$r_{ext} = \sqrt{y} \quad r_{int} = y$$

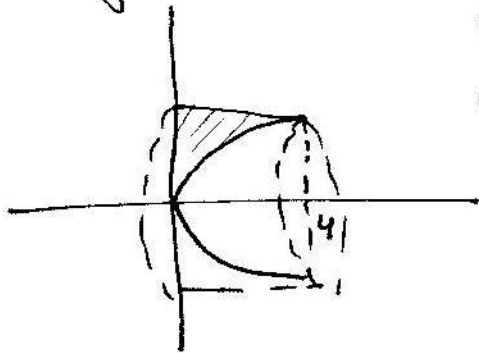
$$V = \int_0^1 \pi (\sqrt{y}^2 - y^2) dy$$

$$V = \int_0^1 \pi (y - y^2) dy \quad V = \pi \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^1$$

$$V = \pi \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{1}{6} \pi$$

5. $y = \sqrt{x}$ $y = 2$ $x = 0$

a) Eje x



$$dV = \pi (r_{ext}^2 - r_{int}^2) dh$$

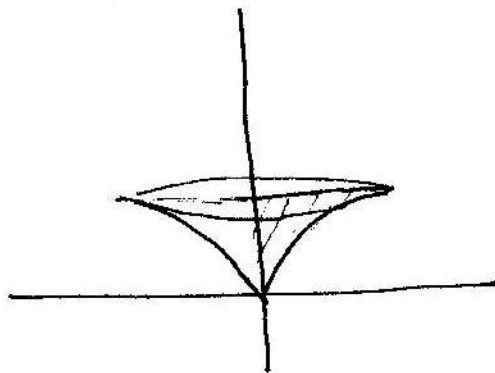
$$r_{ext} = 2 \quad r_{int} = \sqrt{x} \quad dh = dx$$

$$dV = \pi (2^2 - \sqrt{x}^2) dx$$

$$V = \int_0^4 \pi (4 - x) dx \quad V = \pi \left(4x - \frac{x^2}{2} \right) \Big|_0^4$$

$$V = \pi (16 - 8) = 8\pi$$

b) Eje y



$$dV = \pi r^2 dh$$

$$r = y^2$$

$$V = \int_0^2 \pi (y^2)^2 dy \Rightarrow V = \int_0^2 \pi y^4 dy$$

$$V = \frac{\pi y^5}{5} \Big|_0^2 = \frac{32\pi}{5}$$

6 $y = \sqrt{x}$ $y = \frac{x^2}{8}$

Punto de corte.

$$\sqrt{x} = \frac{x^2}{8}$$

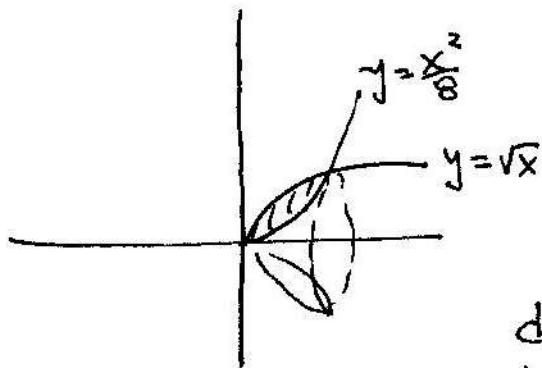
$$x = \frac{x^4}{64}$$

$$x^4 = 64x$$

$$x^4 - 64x = 0 \quad x(x^3 - 64) = 0$$

$$x = 0 \quad x^3 = 64 \quad x = 4$$

Corte (4, 2)



$$dV = \pi (r_{ext}^2 - r_{int}^2) dh$$

$$r_{ext} = \sqrt{x}$$

$$r_{int} = \frac{x^2}{8}$$

$$dV = \pi \left(\sqrt{x}^2 - \left(\frac{x^2}{8} \right)^2 \right) dh$$

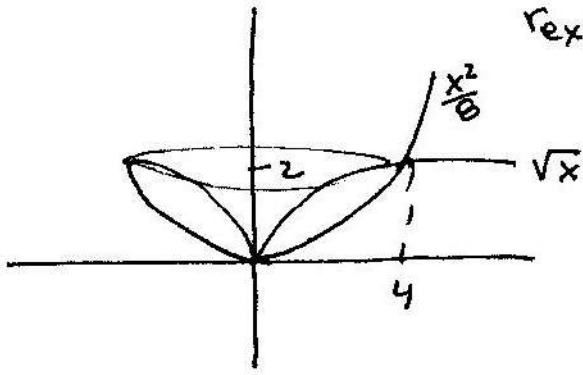
$$V = \pi \left(x - \frac{x^4}{64} \right) dh$$

$$V = \int_0^4 \pi \left(x - \frac{x^4}{64} \right) dx \quad V = \pi \left(\frac{x^2}{2} - \frac{x^5}{5 \cdot 64} \right) \Big|_0^4$$

$$V = \pi \left(\frac{16}{2} - \frac{4^5}{5 \cdot 64} \right)$$

$$V = \pi \left(8 - \frac{4^2}{5} \right) =$$

$$V = \pi \left(8 - \frac{16}{5} \right) = \frac{24}{5} \pi$$

b) alrededor del eje y 

$$r_{ext} = \frac{x^2}{8} \quad r_{int} = \sqrt{x}$$

$$dV = \pi (r_{ext}^2 - r_{int}^2) dh$$

$$dV = \pi \left(\left(\frac{x^2}{8} \right)^2 - (\sqrt{x})^2 \right) dy$$

deben estar los radios en función de "y"

$$y = \frac{x^2}{8}$$

$$y = \sqrt{x}$$

$$8y = x^2 \quad x = \sqrt{8xy}$$

$$y^2 = x$$

$$dV = \pi (\sqrt{8y}^2 - (y^2)^2) dy$$

$$V = \int_0^2 \pi (8y - y^4) dy = \pi \left(\frac{8y^2}{2} - \frac{y^5}{5} \right) \Big|_0^2 = \pi \left(4 \cdot 4 - \frac{2^5}{5} \right)$$

$$V = \pi \left(16 - \frac{32}{5} \right) = \frac{48}{5} \pi$$