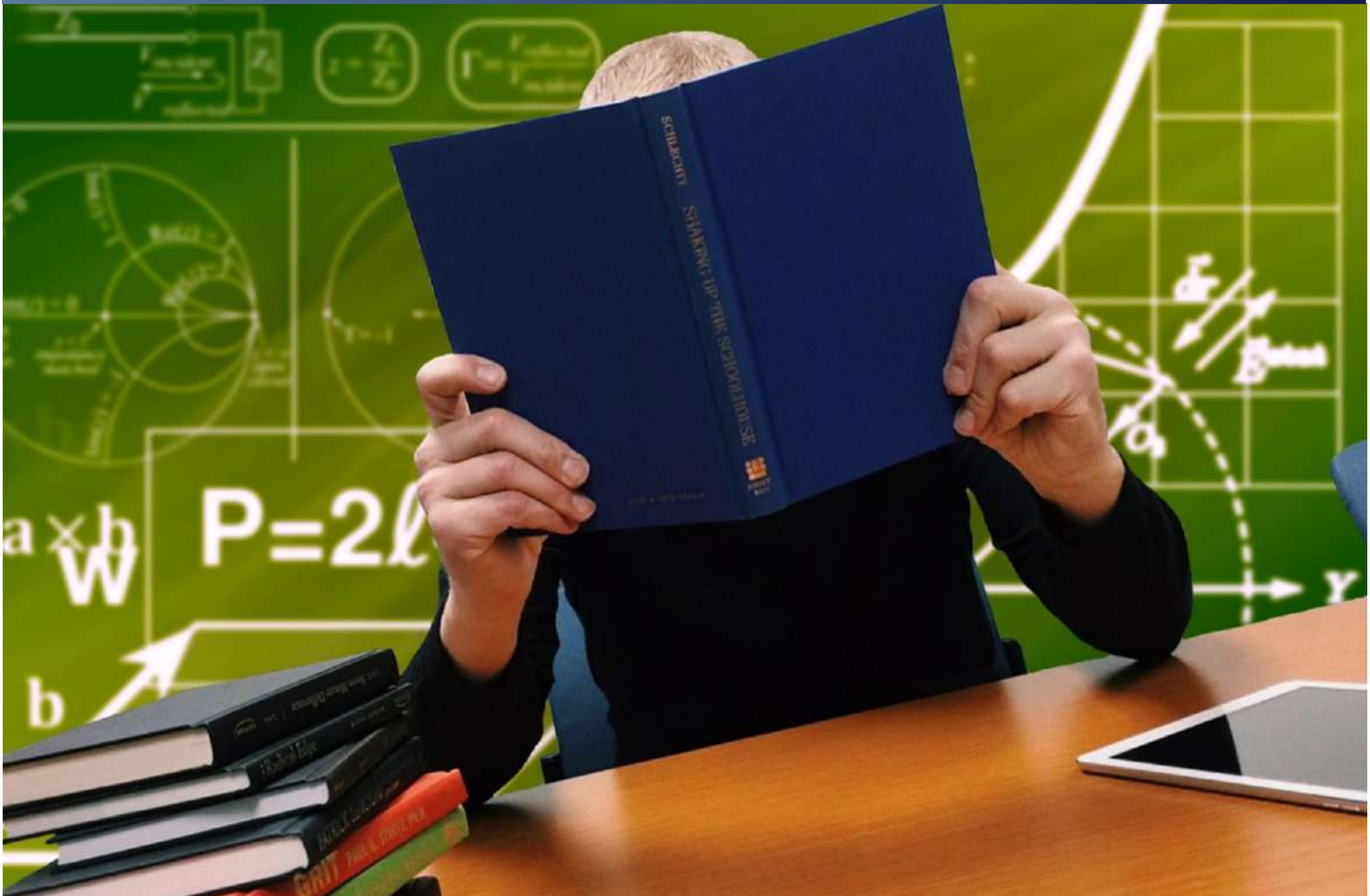
Ejercicios y Talleres



puedes enviarlos a
klasesdematematicasymas@gmail.com

Cálculo Integral de una y varias variables
CB-IV

Tema: Integral definida

Evalúe las siguientes integrales definidas:

1. $\int_{-2}^{0}(3x-6)dx$

2. $\int_{0}^{4}\left(3x-\frac{x^3}{4}\right)dx$

3. $\int_{0}^{\pi}\operatorname{sen}(\mathrm{x})dx$

4. $\int_{-2}^{-1}\left(\frac{2}{x^2}\right)dx$

5. $\int_{-2}^{2}(x^3-2x+3)dx$

6. $\int_{0}^{5}x^{3/2}dx$

7. $\int_{0}^{1}(x^2+\sqrt{x})dx$

8. $\int_{1}^{\sqrt{2}}\left(\frac{s^2+\sqrt{s}}{s^2}\right)ds$

9. $\int_{9}^{4}\left(\frac{1-\sqrt{u}}{\sqrt{u}}\right)du$

10. $\int_{-4}^{4}|x|dx$

11. $\int_{-\sqrt{3}}^{\sqrt{3}}(t+1)(t^2+4)dt$

12. $\int_{1}^{-1}(r+1)^2dr$

13. $\int_{2}^{4}(1-e)dx$

14. $\int_{1}^{95}\frac{x}{\ln(e^x)}dx$

15. $\int_{-2}^{-1}(3w^2-w-1)dw$

16. $\int_{0}^{1}(6\sqrt{x}-x+4)dx$

17. $\int_{1}^{8}\left(x^{1/3}-x^{-1/3}\right)dx$

18. $\int_{0}^{1}e^5\,dx$

19. Evalúe $\left(\int_{1}^{3}x\,dx\right)^3-\int_{1}^{3}x^3dx$

1) $\int_{-2}^{0}(3x-6)\,dx = \frac{3x^2}{2} - 6x \Big|_{-2}^{0} = \left(\frac{3\cdot 0^2}{2} - 6\cdot 0\right) - \left(\frac{3(-2)^2}{2} - 6(-2)\right)$

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$= 0 - \left(\frac{12}{2} + 12\right) = -(6+12) = -18$

2) $\int_{0}^{4}\left(3x - \frac{x^3}{4}\right)dx = \frac{3x^2}{2} - \frac{1}{4}\cdot\frac{x^4}{4}\Big|_{0}^{4} = \left(\frac{3\cdot 4^2}{2} - \frac{1}{4}\cdot\frac{4^4}{4}\right) - \left(\frac{3}{2}0^2 - \frac{1}{4}\cdot\frac{0^4}{4}\right)$

$= \left(\frac{48}{2} - 16\right) - (0) = 24 - 16 = 8$

3) $\int_{0}^{\pi} \operatorname{sen}(x)\,dx = -\cos x \Big|_{0}^{\pi} = -(\cos\pi - \cos 0) = -(-1-1)$

$= -(-2) = 2$

4) $\int_{-2}^{-1}\left(\frac{2}{x^2}\right)dx = \int_{-2}^{-1} 2x^{-2}\,dx = \frac{2x^{-1}}{-1}\Big|_{-2}^{-1} = -\frac{2}{x}\Big|_{-2}^{-1} =$

$\left(\frac{-2}{-1}\right) - \left(\frac{-2}{-2}\right) = 2 - (1) = 1$

5) $\int_{-2}^{2}(x^3 - 2x + 3)\,dx = \frac{x^4}{4} - \frac{2x^2}{2} + 3x \Big|_{-2}^{2} = \left(\frac{2^4}{4} - \frac{2\cdot 2^2}{2} + 3\cdot 2\right) - \left(\frac{(-2)^4}{4} - \frac{2\cdot(-2)^2}{2}\right.$

$\left.+ 3(-2)\right) = \left(\frac{16}{4} - \frac{8}{2} + 6\right) - \left(\frac{16}{4} - \frac{8}{2} - 6\right)$

$= (4 - 4 + 6) - (4 - 4 - 6) = 6 - (-6) = 6 + 6 = 12$

6) $\int_{0}^{5} x^{3/2}\,dx = \frac{x^{5/2}}{\frac{5}{2}}\Big|_{0}^{5} = \frac{2}{5}\, x^{4/2}\cdot x^{1/2}\Big|_{0}^{5} = \frac{2}{5} x^2\cdot x^{1/2}\Big|_{0}^{5}$

$= \frac{2}{5}\, 5^2\cdot 5^{1/2} - \frac{2}{5}\, 0^2\cdot 0^{1/2} = \frac{2}{\cancel{5}}\cdot \overset{5}{\cancel{25}}\cdot\sqrt{5} - 0 = 10\sqrt{5}$

7) $\int_{0}^{1}(x^2 + \sqrt{x})\,dx = \int_{0}^{1}(x^2 + x^{1/2})\,dx = \frac{x^3}{3} + \frac{x^{3/2}}{3/2}\Big|_{0}^{1} = \frac{x^3}{3} + \frac{2}{3}x^{3/2}\Big|_{0}^{1}$

$= \left(\frac{1^3}{3} + \frac{2}{3}1^{3/2}\right) - \left(\frac{0^3}{3} + \frac{2}{3}0^{3/2}\right) = \frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1.$

8) $\displaystyle\int_1^{\sqrt{2}}\left(\frac{s^2+\sqrt{s}}{s^2}\right)ds = \int_1^{\sqrt{2}}\left(\frac{s^2}{s^2}+\frac{\sqrt{s}}{s^2}\right)ds = \int_1^{\sqrt{2}}\left(s+\frac{s^{1/2}}{s^2}\right)ds$ ②

$$= \int_1^{\sqrt{2}}\left(s+s^{-3/2}\right)ds = \frac{s^2}{2}+\frac{s^{-1/2}}{-1/2}\Bigg|_1^{\sqrt{2}} = \frac{s^2}{2}-\frac{2}{s^{1/2}}\Bigg|_1^{\sqrt{2}}$$

$$= \left(\frac{(\sqrt{2})^2}{2}-\frac{2}{(\sqrt{2})^{1/2}}\right)-\left(\frac{1^2}{2}-\frac{2}{1^{1/2}}\right)$$

$$= \frac{2}{2}-\frac{2}{\sqrt[4]{2}}-\frac{1}{2}+2 = \frac{5}{2}-\frac{2}{\sqrt[4]{2}} = \frac{5}{2}-\frac{2}{2^{1/4}}$$

$$= \frac{5}{2}-2^{3/4}$$

9) $\displaystyle\int_9^4\left(\frac{1-\sqrt{u}}{\sqrt{u}}\right)du = \int_9^4\left(\frac{1}{\sqrt{u}}-\frac{\sqrt{u}}{\sqrt{u}}\right)du = \int_9^4\left(u^{-1/2}-1\right)du$

$$\frac{u^{1/2}}{\frac{1}{2}}-u\Bigg|_9^4 = \left(2(4)^{1/2}-4\right)-\left(2\cdot 9^{1/2}-9\right) = (4-4)-(6-9)$$

$$= 0-(-3) = 3$$

10) $\displaystyle\int_{-4}^4|x|dx = \int_{-4}^0 -x\,dx+\int_0^4 x\,dx = -\frac{x^2}{2}\Bigg|_{-4}^0+\frac{x^2}{2}\Bigg|_0^4$

$$= -\frac{0^2}{2}+\frac{(-4)^2}{2}+\frac{4^2}{2}-\frac{0^2}{2} = +\frac{16}{2}+\frac{16}{2} = 8+8 = 16$$

11) $\displaystyle\int_{-\sqrt{13}}^{\sqrt{13}}(t+1)(t^2+4)\,dt = \int_{-\sqrt{13}}^{\sqrt{13}}\left(t^3+4t+t^2+4\right)dt$

$$= \int_{-\sqrt{13}}^{\sqrt{13}}\left(t^3+t^2+4t+4\right)dt = \frac{t^4}{4}+\frac{t^3}{3}+\frac{4t^2}{2}+4t\Bigg|_{-\sqrt{13}}^{\sqrt{13}}$$

$$= \left(\frac{(\sqrt{13})^4}{4}+\frac{(\sqrt{13})^3}{3}+\frac{4(\sqrt{13})^2}{2}+4(\sqrt{13})\right)-\left(\frac{(-\sqrt{13})^4}{4}+\frac{(-\sqrt{13})^3}{3}+\frac{4(-\sqrt{13})^2}{2}\right.$$

$$\left.+4(-\sqrt{13})\right) = \cancel{\frac{169}{4}}+\frac{13}{3}\sqrt{13}+\cancel{26}+4\sqrt{13}-\cancel{\frac{169}{4}}+\frac{13\sqrt{13}}{3}\cancel{-26}+4\sqrt{13}$$

$$= \frac{26}{3}\sqrt{13}+8\sqrt{13}$$

12) $$\int_1^{-1} (r+2)^2 dr = \int_1^{-1} (r^2+4r+4)\,dr = \frac{r^3}{3} + \frac{4r^2}{2} + 4r \Big|_1^{-1}$$ 3

$$= \left[\frac{(-1)^3}{3} + \frac{4(-1)^2}{2} + 4(-1)\right] - \left[\frac{1^3}{3} + \frac{4.1^2}{2} + 4.1\right]$$

$$= -\frac{1}{3} + \cancel{\frac{4}{2}} - 4 - \frac{1}{3} - \cancel{\frac{4}{2}} - 4 = -\frac{2}{3} - 8 = -\frac{26}{3}.$$

13) $$\int_2^4 (1-e)\,dx = x - e\cdot x \Big|_2^4 = (4-4e) - (2-2e)$$

$$= 4 - 4e - 2 + 2e = 2 - 2e = 2(1-e)$$

14) $$\int_1^{95} \frac{x}{\ln(e^x)}\,dx$$ El logaritmo natural y la función exponencial son funciones inversas. Se anulan.

$$\int_1^{95} \frac{x}{1}\cdot dx = \int_1^{95} x\,dx = \frac{x^2}{2}\Big|_1^{95} = \frac{(95)^2}{2} - \frac{1^2}{2} = \frac{9025}{2} - \frac{1}{2}$$

$$= \frac{9024}{2} = 4512$$

16) $$\int_0^1 (6\sqrt{x} - x + 4)\,dx = \int_0^1 (6\cdot x^{1/2} - x + 4)\,dx = \frac{6x^{3/2}}{\frac{3}{2}} - \frac{x^2}{2} + 4x\Big|_0^1$$

$$= \frac{12}{3}x^{3/2} - \frac{x^2}{2} + 4x\Big|_0^1 = \left(4(1)^{3/2} - \frac{1^2}{2} + 4.1\right) - \left(4\cdot 0^{3/2} - \frac{0^2}{2} + 4.0\right)$$

$$= 4 - \frac{1}{2} + 4 - 0 = \frac{15}{2}$$

15) $$\int_{-2}^{-1} (3w^2 - w - 1)\,dw = \frac{3w^3}{3} - \frac{w^2}{2} - w\Big|_{-2}^{-1} = \left[\frac{3}{3}(-1)^3 - \frac{(-1)^2}{2} - (-1)\right] -$$

$$\left[\frac{3}{3}(-2)^3 - \frac{(-2)^2}{2} - (-2)\right] = \left(-1 - \frac{1}{2} + 1\right) - (-8 - 2 + 2)$$

$$= \cancel{-1} - \frac{1}{2} \cancel{+1} + 8 \cancel{+2} \cancel{-2} = \frac{15}{2}.$$

17) $\int_1^8 (x^{1/3} - x^{-1/3})\,dx = \dfrac{x^{4/3}}{\frac{4}{3}} - \dfrac{x^{2/3}}{\frac{2}{3}} \Big|_1^8$ 4

$= \frac{3}{4} x^{4/3} - \frac{3}{2} x^{2/3} \Big|_1^8 = \left(\frac{3}{4}(8^{4/3}) - \frac{3}{2}(8^{2/3})\right) - \left(\frac{3}{4}(1)^{4/3} - \frac{3}{2}(1)^{2/3}\right)$

$= \frac{3}{4} \cdot 16 - \frac{3}{2} = 12 - \frac{3}{2} = \frac{21}{2} = 10.5$

18) $\int_0^1 e^5\,dx = e^5 \cdot x \Big|_0^1 = e^5 \cdot 1 - e^5 \cdot 0 = e^5.$

19) $\left(\int_1^3 x\,dx\right)^3 - \int_1^3 x^3\,dx \Rightarrow \int_1^3 x\,dx = \frac{x^2}{2}\Big|_1^3 = \frac{3^2}{2} - \frac{1^2}{2} = \frac{9}{2} - \frac{1}{2} = \frac{8}{2}$

$= 4$

$\left(\int_1^3 x\,dx\right)^3 = (4)^3 = 64.$

$\int_1^3 x^3\,dx = \frac{x^4}{4}\Big|_1^3 = \frac{3^4}{4} = \frac{81}{4}$

Por tanto $\left(\int_1^3 x\,dx\right)^3 - \int_1^3 x^3\,dx = 64 - \frac{81}{4} = \frac{175}{4}$