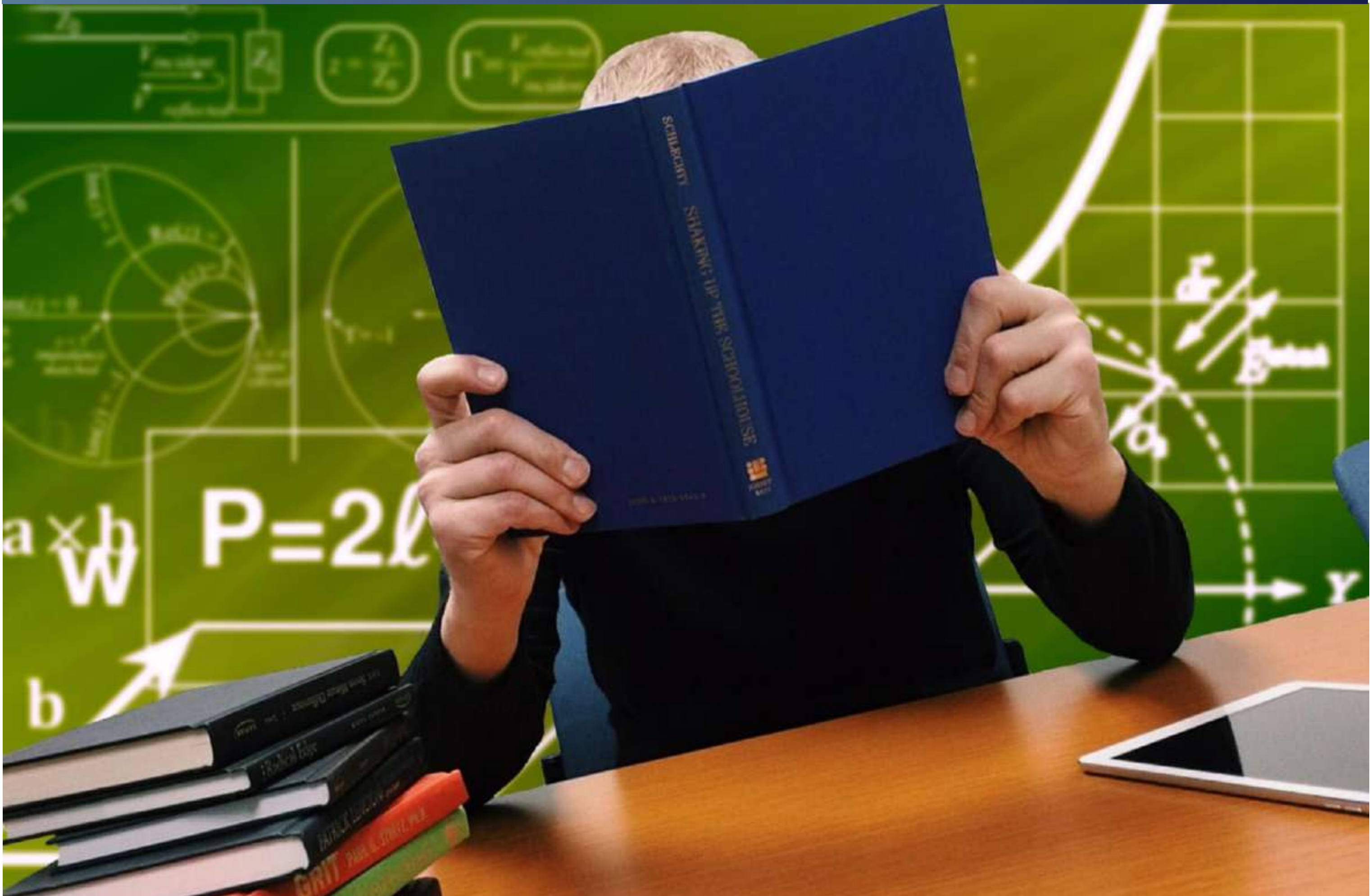
Ejercicios y Talleres



puedes enviarlos a
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1. Utilizando los métodos de solución de las integrales encontrar:

a. $\int x^2 \operatorname{sen} 4x \, dx$ **b.** $\int \frac{x}{1+3x^2} \, dx$

2. Por medio de las propiedades de las integrales resolver:

a. $\int \frac{x^{-3}}{4} \, dx$ **b.** $\int \frac{-x+4+3x^2}{\sqrt{x}} \, dx$

3. Hallar el área limitada por $f(x) = -x^2$ y $g(x) = \frac{1}{2}x - 1$, Graficar.

4. Encontrar los cinco primeros términos de la sucesión y averiguar si converge o no:

$$a_n = \left\{\frac{-n^2+2}{n^2+2n+3}\right\}$$

5. Hallar el volumen del solido formado por la región que gira alrededor del eje x acotado por las gráficas de $y = -x^2 + 3$; $y = x + 1$ realizar la gráfica.

6. Utilizando el cálculo integral hallar el volumen del solido generado por la región acotada por la función $\mathbf{y = x^2}$, $\mathbf{y = 8}$ y $\mathbf{y = 1}$ al girarlo respecto al eje **y**. Dibujar el sólido generado, la unidad de medida es cm, se tendrá en cuenta la precisión en las medidas, utilizar dos decimales donde sea necesario, aproximando matemáticamente.

1. a) $\int x^2 \operatorname{sen} 4x\,dx = -\frac{x^2}{4}\cos 4x + \int \frac{2x}{4}\cos 4x\,dx$

$u = x^2 \quad dv = \operatorname{sen} 4x\,dx$
$du = 2x\,dx \quad v = -\frac{1}{4}\cos 4x\,dx$

$\int x\cos 4x\,dx = \frac{x}{4}\operatorname{sen} 4x - \frac{1}{4}\int \operatorname{sen} 4x\,dx = \frac{x}{4}\operatorname{sen} 4x + \frac{1}{16}\cos 4x$

$u = x \quad dv = \cos 4x$
$du = dx \quad v = \frac{1}{4}\operatorname{sen} 4x$

Por tanto $\int x^2 \operatorname{sen} 4x\,dx = -\frac{x^2}{4}\cos 4x + \frac{1}{2}\left[\frac{x}{4}\operatorname{sen} 4x + \frac{1}{16}\cos 4x\right] + C$

$= \frac{x^2}{4}\cos 4x + \frac{x}{8}\operatorname{sen} 4x + \frac{1}{32}\cos 4x + C$

b) $\int \frac{x}{1+3x^2}\,dx \qquad u = 1+3x^2$
$du = 6x\,dx$
$dx = \frac{du}{6x}$

$\int \frac{x}{u}\cdot\frac{du}{6x} = \frac{1}{6}\int\frac{du}{u} = \frac{1}{6}\ln|u| = \frac{1}{6}\ln|1+3x^2| + C$

2. a) $\int \frac{x^{-3}}{4}dx = \frac{1}{4}\int x^{-3}dx = \frac{1}{4}\cdot\frac{x^{-2}}{-2} = -\frac{1}{8}x^{-2} + C = -\frac{1}{8}\frac{1}{x^2} + C$

b) $\int \frac{-x+4+3x^2}{\sqrt{x}}\,dx = \int \frac{-x+4+3x^2}{x^{1/2}}\,dx$

$= \int (-x^{1/2} + 4x^{-1/2} + 3x^{3/2})dx = -\frac{x^{3/2}}{3/2} + \frac{4x^{1/2}}{1/2} + \frac{3x^{5/2}}{5/2} + C$

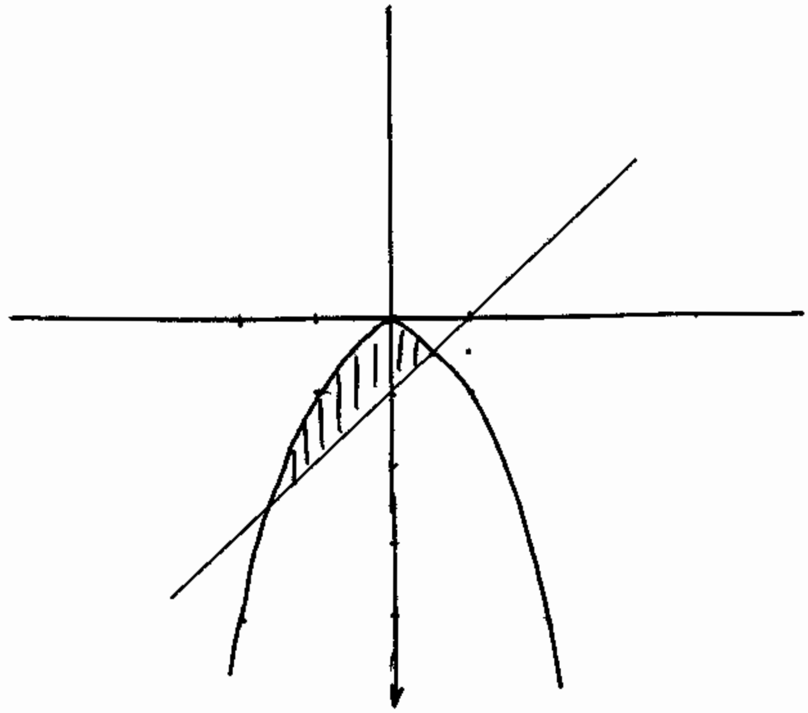
$= -\frac{2}{3}x^{3/2} + 8x^{1/2} + \frac{6}{5}x^{5/2} + C$

3. $f(x) = -x^2 \quad g(x) = \frac{1}{2}x - 1$

Puntos de corte $\quad -x^2 = \frac{1}{2}x - 1 \qquad \frac{1}{2}x - 1 + x^2 = 0$

$x - 2 + 2x^2 = 0 \qquad 2x^2 + x - 2$

$x = \frac{-1 \pm \sqrt{1-4(2)(-2)}}{4} = \frac{-1+\sqrt{17}}{4} \qquad x = 0{,}78 \qquad x = -1{,}28$



$$A = \int_{-1,28}^{0,78} -x^2 - \left(\tfrac{1}{2}x - 1\right) dx = \int_{-1,28}^{0,78} \left(-x^2 - \tfrac{1}{2}x + 1\right) dx$$

$$A = -\frac{x^3}{3} - \frac{1}{2}\frac{x^2}{2} + x \Big|_{-1,28}^{0,78} = -\frac{(0.78)^3}{3} - \frac{1}{4}(0.78)^2 + 0.78 - \left[-\frac{(-1,28)^3}{3} - \frac{1}{4}(-1,28)^2 + (-1,28)\right]$$

$$A = 0,1104 - 0,4075 = \qquad A = 0.4697 - (-0.99)$$

$$A = 1,4597$$

4. $a_n = \left\{ \dfrac{-n^2 + 2}{n^2 + 2n + 3} \right\}$

$$a_1 = \frac{-1^2 + 2}{1^2 + 2\cdot 1 + 3} = \frac{-1 + 2}{1 + 2 + 3} = \frac{1}{6}$$

$$a_2 = \frac{-2^2 + 2}{2^2 + 2\cdot 2 + 3} = \frac{-2}{11}$$

$$a_3 = \frac{-3^2 + 2}{3^2 + 3\cdot 2 + 3} = \frac{-7}{18}$$

$$a_4 = \frac{-4^2 + 2}{4^2 + 2\cdot 4 + 3} = \frac{-14}{27}$$

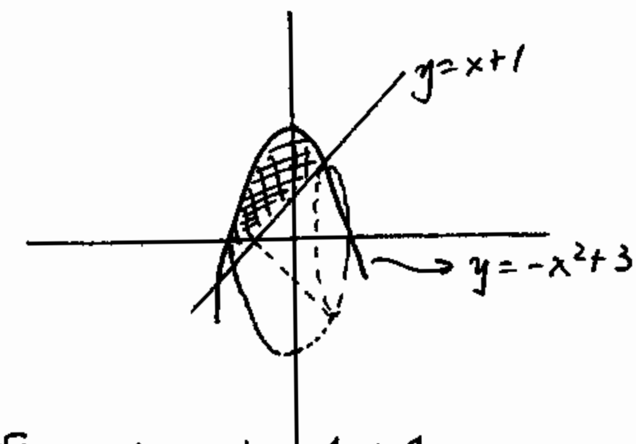
$$a_5 = \frac{-5^2+2}{5^2+2\cdot 5+3} = \frac{-23}{38}$$

$$\lim_{n\to\infty} \frac{-n^2+2}{n^2+2n+3} = \lim_{n\to\infty} \frac{-\frac{n^2}{n^2}+\frac{2}{n^2}}{\frac{n^2}{n^2}+\frac{2n}{n^2}+3} = \frac{-1}{1} = -1$$

Por tanto la sucesión converge.

5. $y=-x^2+3 \qquad y=x+1$

Puntos de corte



$$-x^2+3 = x+1$$
$$-x^2+3-x-1=0$$
$$-x^2-x+2=0$$
$$x^2+x-2=0$$
$$(x+2)(x-1)=0$$
$$x+2=0 \qquad x-1=0$$
$$x=-2 \qquad x=1$$

El sólido se divide en dos de $-\sqrt{3}$ a -1 y de -1 a 1

$$V = V_1 + V_2 \qquad V_1 = \int_{-\sqrt{3}}^{-1} \pi\cdot f(x)^2\,dx = \int_{-\sqrt{3}}^{-1}\pi(-x^2+3)^2\,dx = \pi\int_{-\sqrt{3}}^{-1}(3-x^2)^2\,dx$$

$$V_1 = \pi\int_{-\sqrt{3}}^{-1}(9-6x^2+x^4)\,dx = \pi\left(9x-\frac{6x^3}{3}+\frac{x^5}{5}\right|_{-\sqrt{3}}^{-1} =$$

$$\pi\left[\left(9(-1)-\frac{6(-1)^3}{3}+\frac{(-1)^5}{5}\right)-\left(9(-\sqrt{3})-\frac{6}{3}(-\sqrt{3})^3+\frac{(-\sqrt{3})^5}{5}\right)\right]$$

$$\pi[-7{,}2+8{,}31] = 1{,}11\pi$$

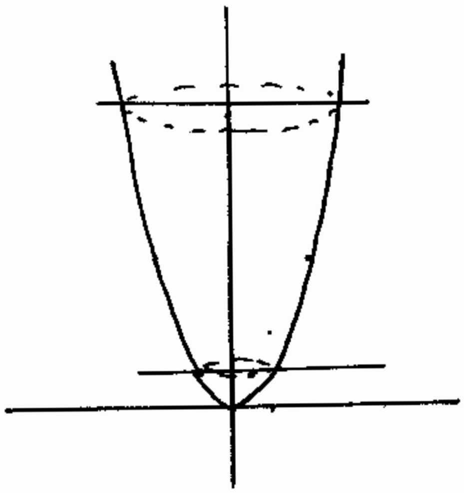
$$V_2 = \pi\int_{-1}^{1}\left((-x^2+3)^2-(x+1)^2\right)dx = \pi\int_{-1}^{1}\left[(9-6x^2+x^4)-(x^2+2x+1)\right]dx$$

$$V_2 = \pi\int_{-1}^{1}(8-7x^2+x^4-2x)\,dx = \pi\left(8x-\frac{7x^3}{3}+\frac{x^4}{4}-\frac{2x^2}{2}\right|_{-1}^{1}$$

$$V_2 = \pi\left[\left(8-\frac{7}{3}+\frac{1}{4}-1\right)-\left(-8+\frac{7}{3}+\frac{1}{4}-1\right)\right] = \pi\left[16-\frac{14}{3}\right] = 11{,}33\pi$$

$$V = V_1+V_2 = 1{,}11\pi+11{,}33\pi = 12{,}44\pi$$

6) $y = x^2 \quad y = 8 \quad y = 1$



$dV = \pi r^2 \cdot dy$

$dV = \pi (x)^2 \cdot dy \qquad x = \sqrt{y}$

$dV = \pi (\sqrt{y})^2 \cdot dy$

$V = \int_1^8 \pi \cdot y\,dy = \frac{\pi y^2}{2} \Big|_1^8 = \frac{\pi}{2}(8^2 - 1^2)$

$V = \frac{\pi}{2}(64-1) = \frac{63\pi}{2}.$